Outline

1. Homeomorphisms
2. Geometric and Algebraic Topology

Equivalent Structures

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- In combinatorics, two graphs are equivalent, if they are isomorphic: there is a bijection between them which preserves the incidence.
- In geometry, two metric spaces are equivalent, if they are isometric: there is a bijection between them which preserves the distance.
- In topology, we want to define equivalence so that equivalent topological spaces $(A, \mathcal{S})$ and $(B, \mathcal{T})$ have the same topological properties, EG connectedness, compactness, Hausdorffity, etc.

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Let $(A, \mathcal{S})$ and $(B, \mathcal{T})$ be topological spaces. A map $h : A \to B$ is said to be open (respectively closed), if $h(U)$ is open (respectively closed) in B for all open (respectively closed) U in A.

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Exercise: give four examples showing that these properties are independent: h can be (open or not open) and (closed or not closed) simultaneously.

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Let $(A, \mathcal{S})$ and $(B, \mathcal{T})$ be topological spaces. A map $h : A \to B$ is a homeomorphism, if it is one-to-one, onto, open and continuous. If such a map exists, we say that $(A, \mathcal{S})$ and $(B, \mathcal{T})$ are homeomorphic.

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Note: continuity of h does not imply continuity of h^{-1} (openness):

Continuity of Inverses

Example 1

$f : [0, 2\pi) \to S^1$ where $t \mapsto (\cos(t), \sin(t))$ is continuous, one-to-one and onto. Its inverse is not continuous at $(1, 0)$.

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To prove that h is open, take an arbitrary open $U \subset A$. Then U' is closed, and hence is compact (Theorem 2 of Lecture 6), as A is compact. Then $h(U')$ is compact in B (Theorem 1 of Lecture 6), and hence is closed (Theorem 3 of Lecture 6), as B is Hausdorff. Then $(h(U'))'$ is open. But h is a bijection, so $(h(U'))' = h(U)$, so $h(U)$ is open, as required. □

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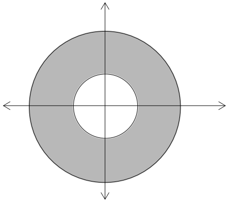
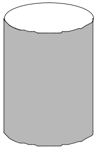
We expect an annulus A in $\mathbb{R}^2$ to be homeomorphic to a cylinder C in $\mathbb{R}^3$ because we can “distort" one into the other.





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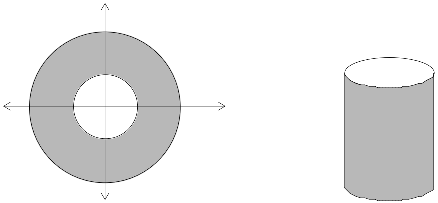

The famous coffee mug and torus homeomorphism is more interesting.

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  3. Discrete topology, half-interval topology and Euclidean topology on $\mathbb{R}$ are pairwise not homeomorphic. In the first one, every point is open, while in the other two not. Half-interval topology is "totally disconnected" – for any two points there are clopen sets containing them; Euclidean topology is connected.

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