A

1. Return your assignment solutions to the MAT4TOP assignment box on level 3, PS2, by 2pm on November 2nd.
2. Please start with the following statement of originality, which must be signed by you: `"This is my own work. I have not copied any of it from anyone else."`
3. The total of this assignment is 20 marks (not counting the Bonus Questions).

1. (Möbius graphs)

By Corollary 2 of Lecture 13, the graphs $K_{3,3}$ and K_5 are non-planar. Show that they are Möbius: find the drawings of $K_{3,3}$ and K_5 on the (open) Möbius band with no edge crossings.

2. (Three Möbius bands)

The surface S_1 is obtained from a closed disc by making three holes and attaching three Möbius bands to them. The surface S_2 is obtained from a closed disc by making two holes and attaching a Möbius bands and a handle to them. Prove that S_1 and S_2 are homeomorphic.

3. (Product of paths)

Prove Lemma 3 of Lecture 15.

4. (Fundamental group of the product)

Prove that for path-connected topological spaces X, Y, the group $\pi_1(X \times Y)$ is isomorphic to $\pi_1(X) \times \pi_1(Y)$.

5. (Fundamental group of the Möbius band)

From Question 4 we can easily find that $\pi_1(\text{torus}) = \mathbb{Z} \times \mathbb{Z}$ and that $\pi_1(\text{cylinder}) = \mathbb{Z}$. Find $\pi_1(\text{Möbius band})$. Hint: a possible first step could be to homotope an arbitrary path to a one whose image lies on the middle line.

Bonus questions

1. (3 marks) Is K_7 toric (can it be drawn on the torus without edge crossings)?

2. (3 marks) Find $\pi_1(\text{projective plane})$. Hint: define and use the action of the group $\mathbb{Z}_2$ on S^2 such that $\mathbb{R}P^2 = S^2/\mathbb{Z}_2$ (in the sense of Question 3 of Assignment 3).